Banking Competition and Economic Stability

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Abstract

We consider a two-period model of a banking system to explore the effects of competition on the stability and efficiency of economic activity. In the model, competing banks lend to entrepreneurs. After entrepreneurs receive the loans for their projects, there is a probability of a shock. The shock implies that a fraction of firms will default and be unable to pay back their loans. This will require banks to use their capital and reserves to pay back depositors, restricting second period lending, thus amplifying the economic effect of the initial shock. There are two possible types of equilibria, a *prudent* equilibrium in which banks do not collapse after the shock, and an *imprudent* equilibrium where banks collapse. We examine the effects of increased competition in this setting.

First, we find existence conditions for prudent equilibria. Second, we show that the effect of increased banking competition is to increase the efficiency of the economy at the expense of increased variance in second period economic results. In particular, if the probability of a shock is small, increased competition raises both expected GDP over the two period and expected activity in the second period, after the shock. Increased competition also increases the attractiveness of *imprudent* equilibria.

Unpredicted regulatory forbearance in the aftermath of a shock can be used to reduce or eliminate the variance in economic activity. However, if regulatory forbearance is expected in response to a shock, the effect on the variance after the shock is ambiguous and can even lead to increased variance after a shock. We also show the expected result that as the size of a shock increases, there is less lending in a prudent equilibrium. Finally we show that independently of the type of equilibria or the possibility of a switch among types of equilibria, increased banking competition increases the amplification effect after a shock.

Keywords: Bank competition, stability, efficiency, forbearance.

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1 Introduction

This paper analyzes a simple two-period model in which a banking system amplifies real economic shocks. We focus on the interaction between the amplification effect and the intensity of competition in the banking sector. A shock affects the economy through the banking channel: an initial systemic shock to productivity leads some firms to default on short term loans, and this weakens banks who must use their equity and reserves to repay short term deposits. This initial reduction in the capital base leads to a reduction in lending in the next period, because of capital adequacy restrictions. Thus the real effects of the initial shock are amplified by the banking system.

We link this effect to competition in the financial market, because in a more competitive market, rates are lower, leading to more borrowing and to increased leverage. As the banking market becomes more competitive, the amplification effect becomes larger, even though the economy is more efficient, so competition creates a tradeoff between efficiency and stability.

There is an extensive literature on the relationship between financial market stability and competition, much of it reviewed in Vives (2010), and which we cover in the next section. Briefly, from the point of view of theory, the predictions are ambiguous. For example, Boyd and Nicoló (2005) note that reduced competition raises interest spreads, which tempts borrowers to choose riskier projects, so the loan book of banks becomes more fragile. On the other hand, in the so called *charter value* approach, a less competitive banking system means that banks are more valuable and owners are less willing to risk them, so they transfer risks to borrowers, see Beck (2008) forr references. Alternatively, with more competition, there are fewer rents from screening and relationship banking (Allen and Gale, 2004), leading to more instability. Beck (2008) shows that there is corroborating empirical evidence for these contrasting arguments.

Note furthermore that there are two kinds of financial fragility: first, fragility leading to bank runs and a second form when the banking system amplifies the effect of an initial real shock, by reducing lending and thus magnifying its economy-wide effects. In this paper we examine the relationship between competition and this second type of fragility using the balance-sheet channel. After an initial economic shock banks need to contract their lending in order to improve their balance sheet, which is weakened by the default of borrowers, in what Tirole (2006) denotes a *credit crunch.* Often the improvement in the balance sheet is required by regulatory authorities, which may even impose more stringent capital adequacy restrictions.[1]

In our model there are two periods. Passive depositors are protected by deposit insurance, which precludes runs.[2] Bank regulation reduces the effects of the associated moral hazard problem by imposing capital adequacy restrictions. At the beginning of the first period, banks lend to firms (think of it as

[1] Recently, Switzerland has imposed a stringent set of capital adequacy rules for Sistemically Important Financial Institutions (SiFis) that will constrain lending by banks. See "Swiss urge capital boost for banks" *Financial Times*, October 4, 2010. `http://www.ft.com/intl/cms/s/0/4a24a1c8-cf26-11df-9be2-00144feab49a.html#axzz2SWZy2mnj`.

[2] This reflects the observation that for SiFis, even large short term deposits are implicitly insured. The Chipre case is the exception, being an example of a hypertrophied banking system.

lending for capital investment) using funds that are provided by short term deposits. At the end of the period, if there is no shock, firms generate revenue to repay loans, and the bank can repay depositors with this income. If there is a shock, some firms are unable to repay their loans. In those cases, the bank have to repay depositors using its own resources (i.e., using its capital and reserves). At the beginning of the second period the firms ask the banks for working capital loans, and the banks lend by obtaining new deposits.

To simplify the analysis of the second period, we assume relationship banking. This means that firms cannot switch banks during the second period, so banks extract all second period profits. In the second period there are no shocks and therefore no risk of failure. This implies that banks always want to lend to every firm in the second period, but they are restricted by capital adequacy restrictions. Thus the second period has no strategic behavior nor risk. All the action occurs in the first period. Note that we differ from much of the literature, which examines the effects of competition on the risk banks by having them choose the risk-return profile of individual bank loans. We study fragility by examining the effects of the interaction between systemic productivity shocks and the intensity of competition on the balance sheet of banks.

In the first period firms are imperfect competitors, and they maximize profits over the two periods, considering the probability of a shock. We model competition via conjectural variations, in order to allow scope for different degrees of competition.[3] Banks start out with some initial capital and can go to the market to request short term funds from depositors. Since deposits are protected by deposit insurance, depositors are always willing providers of funds.

There are many potential entrepreneurs, who own no assets except for the idea of a project. All projects are equally profitable and equally risky. Agents are differentiated by the value of their outside option, which follows a distribution with a continuous density. In the first period, agents whose expected return from the project exceeds their outside option approach banks for loans to carry out their projects. Banks fund entrepreneurs with short period loans which must be returned at the end of each period.

If there is no shock, agents pay back their first period loans, banks pay back depositors without using their capital and reserves and therefore all agents that received a first period loan will also obtain the working capital loan for the second period. However, in the case of a productivity shock, things are different. The shock wipes out the first period returns for a fraction of firms.[4] Those firms are unable to repay the bank and in order to repay depositors, the bank must use its own capital plus any interest on repaid loans. However, since banks must satisfy capital adequacy restrictions, second period lending is restricted in the event of a shock, because the bank's capital and reserves fall after the shock. Thus banks amplify shocks, since by reducing lending, a large number of firms have to cease operations because they have no working capital for the second period. The intensity of the shock is conveniently measured by the fraction of firms unable to repay their first period loan.

[3] We agree that conjectural variations are inconsistent and use them as a convenient way of mapping the effects of different competitive assumptions, by using different values of a single parameter. See (Dixit, 1986) for a defense of this approach.

[4] Though they remain viable for the second period if the bank can fund them.

In principle, there is another possible outcome, which occurs when banks are overextended and repayment of deposits in case of a shock wipes out the capital and reserves of the bank (leading to an *imprudent* equilibrium). In that case there is a collapse of the banking system and no second period lending.[5] In the *prudent* equilibrium, banks are judicious and choose to lend quantities that even in the case of a shock, will allow them to survive. In the *imprudent* equilibria, banks are improvident in the sense that they choose to lend larger amounts than when behaving prudently, and if the shock happens, the banking system collapses.

When capital adequacy restrictions are loose, so that in the *prudent* equilibria under a shock banks are close to bankruptcy, they choose the *imprudent* equilibria, requiring the need for prudential regulation to avoid these outcomes. The banking regulator can exclude *imprudent* equilibria through judicious use of capital adequacy restrictions but an inappropriate application of these conditions, or the (correct) belief that these may be loosened in case of a negative real shock, may lead to imprudent equilibria.

Note that the correct application of capital adequacy regulations, by precluding the collapse of the banking system, implies that there is no need for deposit insurance, so providing it is costless to society. However, even without a banking system collapse, increased competition leads to increased variance in economic outcomes. Basically, as competition increases, banks charge a lower interest rate and lend more. In the case of no shock, there is more economic activity. On the other hand, when there is a crisis, a larger mass of entrepreneurs fail to pay their loans, leading to a larger reduction in bank capital. This, in turn, reduces second period lending. Hence, second period activity is more variable as competition increases.

We also examine the effect of capital adequacy rules. In response to a shock, governments usually relax the capital adequacy rules, at least in the short run. This reduces the magnitude of the shock. We show that this emergency response works only when banks do not expect the rule to be relaxed. If banks have perfect foresight about the future capital adequacy rule, this is incorporated in their lending decisions. Hence the effects of an expected future relaxation in the capital adequacy rules in response to a shock are ambiguous. The effects on on pre-shock lending and therefore on the variance of post-shock GDP depend on specific parameter values.

The main result of this paper is that independently of the type of competition, and even considering the possible switch from a *prudent* to an *imprudent* equilibrium, an increase in the degree of competition in the banking sector increases the variance of post shock activity and hence the variance of GDP.

One final issue is that the model is not a general equilibrium model, in the sense that it relies on the existence of deposit insurance and the supply of deposits is perfectly elastic. Using deposit insurance is not uncommon in the literature, as in Allen and Gale (2004) and others. That paper does, however, include a cost of insurance to banks that is independent of individual riskiness, but which covers the aggregate cost of deposit insurance. In our model, when the prudential equilibrium is chosen and there is no banking collapse, the real cost of insurance is zero. Our model could be adapted to accommodate a flat insurance rate, with no change in the main results, and with additional difficulties, to an increasing

[5] Because of deposit insurance, there is no possibility of a bank run a la Diamond and Dybvig (1983). Since all banks are assumed identical, symmetry implies that there is a simultaneous banking collapse.

supply schedule for deposits. However, guaranteeing that the insurance rate is actuarially fair would complicate the calculations.

2 Literature review

There is a large literature on the relationship between stability and competition in financial markets, so this review will highlight important contributions, under the proviso that it will leave many relevant papers unmentioned. As mentioned above, Vives (2010) provides a comprehensive review of the empirical and theoretical literature.

On the theoretical side, Allen and Gale (2004), following an earlier tradition, use a static model to study the relationship between competition and stability. In their model, firms can choose the return and riskiness of loans, and competition leads to riskier lending. This also implies that efficiency is not attained with competitive markets, because of excessive risk taking.

There is an alternative literature, which follows from Stiglitz and Weiss (1981) in which the risk and interest rates charged on loans are positively related. Hence, as in Boyd and Nicoló (2005), the risk taking behavior of borrowers increases as interest rates go up due to less intense competition. This view is further explored in De Nicolo, Boyd, and Jalal (2009), which uses an elegant model that includes a safe asset to make the point that there is no one-to-one relationship between stability and competition. Hakenes and Schnabel (2011) make a similar point, in a model in which banks have equity, so that a capital adequacy ratio can be used to regulate risk taking. The authors obtain the surprising result that limiting the leverage of banks increases entrepreneurial risk taking, since less competition (due to a higher capital adequacy ratio) translates into higher interest rates on loans. Very recently, Carletti and Leonello (2012) describe a two period model where competition leads to increased stability because when there is competition, banks profits from lending are low and keeping large reserves is cheap. so banks do not default. With less competition they obtain a mixed equilibrium with some banks choosing a risky strategy and others choosing a safe strategy. Hence the banking system is less stable as competition decreases.

In a recent paper, Martinez-Miera and Repullo (2010) show that the results of Boyd and Nicoló (2005) depend crucially on having perfect correlation of loan defaults. They note that when loans are not perfectly correlated, more competition reduces the return on loans that do not default, so the total effect of competition on stability depends not only on the reduced riskiness of loans but also on the reduced margin on loans that do not default. Using an imperfectly competitive model, Martinez-Miera and Repullo (2010) establish that the second effect is dominant under perfect competition and that in less competitive markets there is a U-shared relationship between competition and stability. In our model, only the margin effect is present.

Wagner (2010) uses an alternative argument to the same purpose by noting that even though increased competition leads to lower rates and therefore to borrowers that choose less risky projects, the banks can also influence the level of risk of their loans. When facing lower return due to competition, they will choose borrowers with riskier projects and higher returns, and this will counteract the stabilizing effect of competition of Boyd and Nicoló (2005).

A recent unpublished monograph by Freixas and Ma (2012) develops a more tractable model that obtains the results of Martinez-Miera and Repullo (2010) but incorporates the possibility of bank runs of the Diamond and Dybvig (1983) type.[6] They have banks that use two types of funding: insured deposits or uninsured money market funds. They differentiate between portfolio, liquidity and solvency risk and show that the conditions under which competition reduces risk depends on a simple condition involving the fraction of insured deposits in bank liabilities, the productivity of projects and the interest rate. When productivity is low and banks are funded with insured deposits, competition increases total credit risk. They argue that their more detailed model allows them to interpret the different results obtained in the empirical literature, which they review in detail.

The empirical evidence is mixed. Early studies of the effects of bank liberalization in the US Keeley (1990), Edwards and Mishkin (1995) and others showed that liberalization lowered the charter values of banks and this increased risk taking. For Spain, Saurina-Salas et al. (2007) found that liberalization and increased competition was associated to higher risk, measured as loan losses to total loans.

In cross country studies, diverse studies show that increase competition contributes to stability. This is the case of Schaeck, Cihak, and Wolfe (2009), who use the Panzar and Rosse H-statistic to study the probability of a crisis using 41 countries. They also point out that bank concentration is associated to higher probability of crisis, so concentration and competition capture different aspects of the fragility of banking systems. Similarly, in a recent working paper, Anginer, Demirguc-Kunt, and Zhu (2012) use a sample of 63 countries to look at the effects of competition (measured by the Lerner index). They incorporate the (co-)dependency among bank risks, in order to examine systemic financial fragility, rather than at the level of individual banks. They find a stabilizing effect of competition. On the other hand, in a recent article Beck, Jonghe, and Schepens (2013), who incorporate the regulatory framework and financial market characteristics as an explanatory variable in the country cross sections, find a positive association between market power and a measure of financial fragility.

After reviewing the evidence, Vives (2010) concludes:

> "Theory and empirics point to the existence of a trade-off between competition and stability along some dimensiones. Indeed, runs happen independently of the level of competition but more competitive pressure worsens the coordination problem of investors/depositors and increases potential instability, the probability of a crisis and the impact of bad news on fundamentals."

[6]Even in the case of bank runs there are two different approaches: the multiple equilibria-sunspot view of Diamond and Dybvig (1983) (the expectation of a collapse, coupled to the maturity mismatch leads to runs) and those where runs are triggered by the deterioration of fundamentals. There is an alternative approach to the same problem, as for example in Rochet and Vives (2004), the bank fails because the fundamentals are weak and this leads to a higher probability of a run.

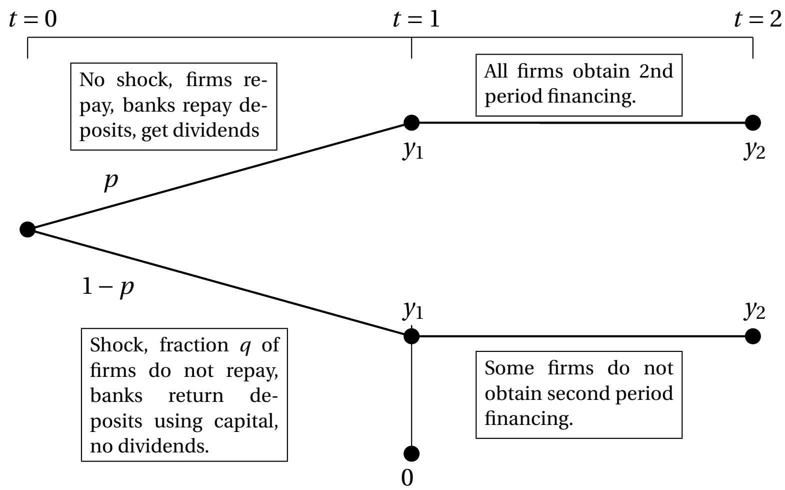

Figure 1: Scheme of returns over time.

3 The basic model

We consider an economy with three dates ($t = 0, 1$ and 2) and two periods. There is a continuum of risk neutral entrepreneurs with zero assets, where we denote an entrepreneur by $z \in [0, 1]$. In the first date, $t = 0$, agents decide whether to undertake a risky project which lasts two periods, or to exercise an outside option. Although the risky project is the same for all entrepreneurs, these agents are differentiated by the value of their outside option, which yields a safe return u_z for entrepreneur z at the end of the second period.[7] The distribution of u_z is given by $G(\cdot)$, which has a continuous density $g(\cdot)$ and full support $[0, U]$.

The risky project requires one unit of investment capital at $t = 0$ that the agent must borrow from a bank. The project provides returns at $t = 1$ and at $t = 2$. The returns in $t = 1$ depend on the state of the economy, denoted by s, which s can take two values, high (h) and low (l). In state h, which occurs with probability p, the economy has a high productivity shock, i.e., all projects are successful, in which case they return y_1. In state l, the economy suffers a low productivity shock, in which case each project succeeds and returns y_1 with probability $q \in (0, 1)$ and fails (returns 0) otherwise. Figure 1 shows the events over time:

At $t = 1$ all firms (even those that were unsuccessful) can apply for a working capital loan λ from banks, in order to operate in the second period.[8] In the second period there are no shocks and all firms that obtain the working capital loan receive a return of y_2 at the end of the period. The following timeline shows the relevant variables at the different points in time:

The economy has two other clases of risk neutral agents: depositors and banks. Depositors lend

[7] This is similar to the assumption in Boyd and Nicoló (2005) and used for the same purpose: to differentiate among entrepreneurs and thus obtain a demand curve for loans.

[8] An alternative is to allow only successful firms to be able to ask for loans, and we have examined this case. It is more complex, because failure will affect both the demand and the supply of loans, whereas in the present case, only loan supply is affected. On the other hand, we believe our formulation is reasonable if we interpret the first loan as one of initial investment plus working capital and the second one as a loan of working capital and for maintenance costs.

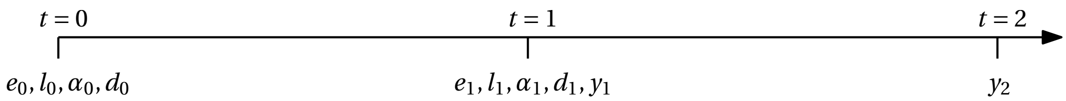

Figure 2: Timeline of the model showing the active variables and parameters.

to banks each period and receive their money back at the end of the period. Their supply is perfectly elastic at a risk-free rate that we normalize to zero, for notational simplicity. Depositors do not ask for more than the risk-free rate because the government insures deposits at failed banks. This implies that depositors play a passive role in the model.[9]

Define $\beta = (1 + \rho)^{-1}$ as the discount factor associated with the cost of capital ρ. We need the following assumption:

Assumption 1 $\beta p y_1 - 1 > 0$,

Assumption 1 implies that the expected net present value of the second stage of a project is positive, even in the state of nature $s = l$.

Banks are the financial intermediaries of this economy, specialized in channeling funds from investors to entrepreneurs. There are N identical banks. To fund their projects, entrepreneurs borrow from banks, and banks compete to attract entrepreneurs. At date t, each bank extends loans l_t that are financed by deposits d_t and inside equity e_t. Hence the budget constraint for a representative bank at date t is:

$$l_t \leq d_t + e_t \tag{1}$$

Each bank is run by a single owner-manager who provides the equity e_t; the owner's opportunity cost of capital is $\rho > 0$, so that equity financing is more expensive than deposit financing. This assumption is typically assumed in the literature.[10]

Banks compete for borrowers in the first stage and have an ongoing relationship with the borrower in the second period, so they can extract all rents from borrowers in the second stage. We assume that entrepreneurs do not use internal financing in period 2.[11] Also, following Repullo and Suarez (2009), we assume that it is impossible to recapitalize a bank at date $t = 1$. Their argument, which we adopt, is that the dilution costs of an urgent equity raise could be high for a bank with opaque assets in place.

Finally, there is a financial regulator that imposes capital adequacy requirements that limit the

[9]At this stage, this assumption is for simplicity only. As we will see later, banks behave prudently and never fail. Thus there is no need for insurance, and depositors do not face systemic risk.

[10]See Berger and Ofek (1995) for a discussion of this issue; and Gorton and Winton (2003), Hellmann et al. (2000) and Repullo (2004) for a similar assumption.

[11]This simplification is standard in relationship-banking models; see, for example, Sharpe (1990) or von Thadden (2004). Moreover, if entrepreneurs' first-period profits are small relative to the amount of the working capital loan, the effects of relaxing this assumption would be negligible (Repullo (2012)).

amounts that banks can lend to fixed multiple of their capital. For banks, this implies that:

$$l_t \leq \frac{e_t}{\alpha_t}, \quad 0 \leq \alpha_t \leq 1, t - 0, 1 \tag{2}$$

where α_t is the capital adequacy requirement in period t. In general,

We assume that $\alpha_t,\ t = 0,1$ are fixed by the regulator. We consider two cases, one in which their value is fixed at $t = 0$ and that banks, entrepreneurs and depositors believe that this parameter will not be reset at $t = 1$, independently of the state of nature in that period. The second case is when banks internalize the belief that the value of the parameter will change at $t = 1$ in case of a shock, the only situation in which a change in this parameter makes a difference.

4 Equilibrium

Since this is a two period model, we solve it by backwards induction.

4.1 Equilibrium at $t = 1$

We assume, at seems reasonable, that only in the state of the nature l there is the possibility of a *credit crunch*[12], in the sense that some profitable projects cannot get financing –even with no uncertainty about their profitability– because banks do not have enough equity (capital plus reserves) to finance them, given the capital adequacy restriction. When the state of the nature is h, all projects succeed and managers pay back their first period loans. Thus banks have enough capital and reserves, after returning the deposits, to finance all applications for loans in period 1, and all agents know this.[13]

Suppose that the realized state of nature was s. At date $t = 1$, the following variables are taken as given by agents: (1) the equilibrium interest rate charged on first period loans, r_0; (2) the banks' capital in the state s, e_1^s; (3) the total amount of credit given by the representative bank to finance projects in the first period, l_0; (4) the number of entrepreneurs that obtained funding in the first period, $G(\overline{u})$, where $\bar{u}$ is the utility cutoff for entrepreneurs.

Entrepreneurs, even if their projects fails in the first period, can ask for a loan of amount λ from the same bank from which they asked their original loan, because of our assumption of an *ongoing relationship*.[14] Given that firms cannot apply for loans from other banks, the incumbent bank can extract all the rents from its borrowers at this stage, and the entrepreneurs' participation constraint

[12] A *credit crunch* is defined as a situation in which there is a reduction in the general availability of credit or a sudden tightening of the conditions required to obtain a loan from the banks.

[13] We can always adjust the parameters of the model –in particular, the magnitude of the shock–to have this case in state l. That is, we do not examine the case of a capital constrained banking system.

[14] It is possible to consider as an alternative assumption that firms that fail in the first period go out of the market, and we have also examined this case (which has the added complication that in the bad state, both the demand and the supply of loans depends on the fraction $1 - q$ of failing firms). However, in an interpretation of the original investment as including initial investment plus working capital, and a second period in which only working capital is needed, because the project is not a failure but has not met initial expectations, the interpretation we include is more appropriate.

will be binding in equilibrium.[15] This means that banks cannot expect failed entrepreneurs to pay a penalty fee for having defaulted on their first period loans. Hence, if entrepreneur z gets financing, we obtain the interest rate charged to that entrepreneur:

$$y_2 - (1 + r_1)\lambda = 0 \Rightarrow 1 + r_1 = \frac{y_2}{\lambda} \tag{3}$$

As the second stage of the project is profitable for banks, banks will want to finance the maximum number of these projects they can. The demand for second period loans is λl_0, where we have used the fact that first period loans are of size 1. Thus, in state s, the bank solves the following problem at $t = 1$:

$$\begin{aligned} \underset{\{l_1^s, d_1^s, Div^s\}}{\text{Max}} \; & \beta\left[(1 + r_1)l_1^s - d_1^s\right] + Div^s \\ \text{s.t.} \quad & e_1^s + d_1^s - l_1^s = Div^s \\ & e_1^s - Div^s \geq \alpha_1 l_1^s \\ & \lambda l_0 \geq l_1^s \\ & Div^s, l_1^s, d_1^s \geq 0 \end{aligned}$$

For each bank and in each state $s = h, l$, the decision variables are the total amount of credit to provide, l_1^s, the total amount of deposits to raise, d_1^s and the first period dividends policy, Div^s. The objective function is the discounted utility of the representative bank at date $t = 1$, and it consists of two terms. The first term $\beta\left[(1 + r_1)l_1^s - d_1^s\right]$ is the net present value of the bank's net profits of date $t = 2$, where $(1 + r_1)$ is determined as in (3). The second term Div^s is the cash left over, which is used to pay dividends to shareholders at $t = 1$.

The first (equality) restriction is the time $t = 1$ budgetary restriction of the bank. The second restriction is the capital adequacy restriction, which applies after dividends are paid and determines the loanable funds. The third restriction requires that total loan supply must be smaller than loan demand in each state (otherwise the cost of loans is zero). The following proposition characterizes the equilibrium at date $t = 1$:

Proposition 1 *At date $t = 1$, each bank makes loans of $l_1^s = \min\left\{\frac{e_1^s}{\alpha_1}, \lambda l_0\right\}$, takes deposits of $d_1^s = (1 - \alpha_1)l_1^s$ and pays dividends $Div^s = e_1^s - \alpha_1 l_1^s$, $s = h, l$.*

Proof: See Appendix. ∎

This result links the stock of capital of banks at $t = 1$, e_1^s, to the supply of credit that each bank provides for the second period, l_1^s. In particular, note that if the capital e_1^s is sufficiently low there will be a *credit crunch*, as banks will not be capable of meeting the effective demand for loans, λl_0. This

[15] This is not essential; the borrower could split the second period surplus with the bank, the division of the surplus reflecting the ease of substitution with other banks. However, including this possibility would have added a parameter to the model without materially changing our results.

happens because the capital adequacy restriction limits the quantity of credit that banks can supply, and this may result in an unmet demand for credit given by $\mathrm{Max}\left\{\lambda l_0 - \frac{e_1^s}{\alpha_1}, 0\right\}$. Notice that there is a credit crunch only in state l if $e_1^l < \lambda\alpha_1 l_0 \leq e_1^h$, i.e., if banks do not have enough internal capital in that state. Note also that an unexpected lowering of the capital adequacy ratios at $t = 1$ in the bad state of the world can eliminate the credit crunch. However, as we will see in section 6.1 below, forbearance sis guaranteed to be effective only occurs if the change is unexpected.[16]

Corollary 1 *If $e_1^s > 0$, a regulator can always eliminate a credit crunch by lowering the value of the capital adequacy ratio α_1 from the value expected by economic agents.*

Now we study the determination of e_1^s. In the state of nature $s = h$, we know that all projects succeed, so each entrepreneur has the resources to pay his debt at the end of the first period. Therefore, the capital of the representative bank at date $t = 1$ is (before paying dividends):

$$e_1^h = (1 + r_0)l_0 - d_0 \tag{4}$$

where $d_0 = l_0 - e_0$ are the deposits that the bank must repay at the end of the first period, just before $t = 1$. As we have assumed that in the state h the representative bank has enough capital to finance all the entrepreneurs who ask for a loan, i.e., there is no *credit crunch*, then it must hold that $\alpha_1 l_1^h = \lambda\alpha_1 l_0 \leq e_1^h$.

Using the results of Proposition 1, we obtain the net present value of the bank at date $t = 1$ when the state of the nature is $s = h$:

$$\begin{aligned}\Pi_1^h &= \beta\left[(1 + r_1)l_1^h - d_1^h\right] + \left[e_1^h + d_1^h - l_1^h\right] \\ &= \left[r_0 + e_0/l_0 + \beta(y_2 - \lambda) - \lambda\alpha_1(1 - \beta)\right] l_0 \end{aligned} \tag{5}$$

where we have used the interest rate r_1 obtained in equation (3), (4) and the results of proposition 1.

Now we study the case when the state of the world is $s = l$. Recall that in this case, from the point of view of $t = 0$, each project succeeds with probability q and fails with probability $1 - q$. By the Law of Large Numbers, exactly a fraction $1 - q$ of entrepreneurs fail, so in this economy there is no aggregate uncertainty. Therefore, ql_0 entrepreneurs succeed and pay their debts. On the other hand, we have assumed that all agents, including those who fail in the first period and are unable to repay their loans,

[16] Temporary forbearance of capital adequacy strictures is common when banks are distressed. For instance, in Mitra, Selowsky and Zalduendo, "Turmoil at Twenty: Recession, Recovery and Reform in Central and Eastern Europe and the Former Soviet Union", World Bank 2010. we find:

> "Some previous episodes of systemic banking distress, such as Argentina 2001, Bulgaria 1996, Ecuador 1999, Indonesia 1997, Korea 1997, Malaysia 1997, Mexico 1994, the Russian Federation 1998, and Thailand 1997 have also seen regulatory forbearance. Specifically, to help banks recognize losses and allow corporate and household restructuring to go forward, the *government might exercise forbearance* either on loss recognition, which gives banks more time to reduce their capital to reflect losses, or *on capital adequacy, which requires full provisioning but allows banks to operate for some time with less capital than prudential regulations require*."

ask for a working capital loan to continue their projects in the second period. Now, the capital of the representative bank at date $t = 1$ is:

$$e_1^l = \text{Max}\{q(1+r_0)l_0 - (l_0 - e_0), 0\} \tag{6}$$

where the Max operator arises due to limited liability of banks. Recall that if this capital is zero then the bank fails at date $t = 1$, and its depositors are paid with the residual value of the bank ($q(1+r_0)l_0$) plus the compensation made by the government from deposit insurance. This happens when the probability of success q satisfies:

$$q < \widehat{q} \equiv \frac{1 - (e_0/l_0)}{1 + r_0} \tag{7}$$

If the fraction of firms that manage to repay after the shock are $q \geq \widehat{q}$, then banks do not fail in the event of a crash, though their second period capital shrinks. As in the previous case, banks always want to finance as many projects as possible. As we have mentioned before, to make things interesting, we assume that in state l banks cannot finance all the entrepreneurs who ask for a second period loan, so that[17]

$$l_1^l = \frac{e_1}{\alpha_1} < \lambda l_0 \tag{8}$$

From the discussion above, only a fraction $\theta \in [0, 1)$ of the demand for credit λl_0 is going to be satisfied:

$$\theta = \frac{l_1^l}{\lambda l_0} = \frac{q(1+r_0) - (1 - (e_0/l_0))}{\lambda \alpha_1} \tag{9}$$

The variable θ measures the ratio of the second period economy under a shock to the size of the economy without the shock, i.e., when it is close to one, the economy is able to resist the shock without many ill effects. Similarly, $1 - \theta$ is the fraction of entrepreneurs rationed by banks at date $t = 1$, and can be interpreted as the magnitude of the *credit crunch*; and $1 - q$ can be interpreted as the *magnitude of the shock*, as it represents the fraction of entrepreneurs that cannot repay their loans, For further reference note that $\text{Var}(l_1) = p(1-p)(\lambda l_0 - l_1^l)^2 = p(1-p)(1-\theta)^2(\lambda l_0)^2$.

A final observation: as a consequence of proposition 1, in the case of a shock banks do not pay dividends in the first period because reinvesting all repayments into loan renewals is more profitable.

The discounted utility of the representative bank at $t = 1$, after a shock can be written as:

$$\begin{aligned} \Pi_1^l &= \beta\left[(1+r_1)l_1^l - d_1^l\right] + \overbrace{Div^l}^{=0} \\ &= \frac{\beta}{\alpha_1}\left[\frac{y_2}{\lambda} - (1-\alpha_1)\right] \text{Max}\{q(1+r_0)l_0 - (l_0 - e_0), 0\} \end{aligned} \tag{10}$$

which should be compared to the profits at $t = 1$ in the case of no shock, given by equation (5). The next step is to proceed to the analysis of the profit maximization problem at date $t = 0$.

[17] Otherwise the case with a shock can be treated as if it were the case without a shock and nothing happens after the shock.

4.2 Equilibrium at $t = 0$

In the first period, given an aggregate demand for loans $L(r_0)$, banks choose the profit-maximizing volumes of deposits (d_0), equity (e_0), and loans (l_0). This automatically defines the equilibrium interest rate r_0 charged to entrepreneurs.

4.2.1 The demand for credit

Given the first period interest rate r_0 charged by banks, we define $\overline{u}(r_0) \equiv [p + (1-p)q][y_1 - (1 + r_0)]$ as the expected net future value (at the end of the second period) that the entrepreneur will obtain if he undertakes the two-stage risky project. Observe that the entrepreneur gets no rents from operating the firm in the second period because the banks extract all profits. An entrepreneur z will be willing to embark in this venture rather than stay with the safe option only if $\overline{u}(r_0) \geq u_z$. These participation constraints implicitly define an aggregate loan demand that is decreasing in the interest rate at $t = 0$, given by:

$$L(r_0) = \int_0^{\overline{u}(r_0)} g(u)du = G\left([p + (1-p)q][y_1 - (1 + r_0)]\right) \tag{11}$$

with $\frac{\partial L(r_0)}{\partial r_0} = g(\bar{u}(r_0))\frac{\partial \bar{u}(r_0)}{\partial r_0} < 0$. As usual, it will be more convenient to work with the inverse demand function, $r_0(L)$. We can rearrange the last equation to obtain:

$$1 + r_0(L) = y_1 - \frac{G^{-1}(L)}{p + (1-p)q} \tag{12}$$

where $\sum_{j=1}^{j=N} L_j \equiv L$. This expression defines explicitly a downward sloping inverse demand of loans. We make the following standard assumption:

Assumption 2 *The distribution function of outside options $G(z)$ is twice-continuously differentiable, positive, and concave for all $L \in (0,1)$*

4.2.2 The banks' optimization problem

At the beginning of the first period, each bank chooses the volume of its deposits (d_0), equity (e_0), and loans (l_0). Given the balance sheet identity, $l_0 = d_0 + e_0$, only two of these variables can be chosen independently. Recalling that α_0 is the capital adequacy constraint at $t = 0$, the representative bank at $t = 0$ solves:

$$\begin{aligned} \underset{\{l_0\}}{\text{Max}}\, \Pi_0 &\equiv \beta\left[p\Pi_1^h + (1-p)\Pi_1^l\right] - e_0 \\ \text{s.t.} \quad l_0 &\leq (e_0/\alpha_0) \end{aligned} \tag{13}$$

where the objective function is the expected net present value of profits of the bank while the restriction corresponds to the capital adequacy condition at $t = 0$.

Recall from the comments on equation (7) that if $q < \widehat{q}$, banks go bankrupt in the low state ($e_1^l = 0$). In that case, $\Pi_1^l = 0$ and there are positive profits only in the good state ($\Pi_1^h > 0$). Noting from the definition of $\hat{q}$ that a reduction in l_0 leads to a reduction in $\hat{q}$, banks, by lending less could have remained solvent and thus would maximize over both the good and bad states of the world.[18] This corresponds to what we denote by *prudent* behavior, leading to a symmetric *prudent equilibrium*. Conversely, behavior leading to bankrupt bank is *imprudent* behavior, and leads to an *imprudent equilibrium*. This is the type of equilibrium behavior in which banks could be accused of "privatization of profits and socialization of losses".

Hence, there are two different expressions for the profit function, depending on whether $q > \widehat{q}$, and banks survive the shock, and the case in which the inequality is reversed and banks fail. Thus we define two functions associated to profits in the two states:

$$\begin{aligned}
\Omega^h(L) &= \beta p \left(\left(y_1 - \frac{G^{-1}(L)}{p + (1-p)q} \right) - 1 + \beta(y_2 - \lambda) - \lambda\alpha_1(1-\beta) \right) \\
\Omega^l(L) &= \frac{\beta^2(1-p)}{\alpha_1} \left(\frac{y_2}{\lambda} - (1-\alpha_1) \right) \left(q \left(y_1 - \frac{G^{-1}(L)}{p + (1-p)q} \right) - 1 \right)
\end{aligned} \tag{14}$$

Profits at $t = 0$ depend on whether the bank fails in period 1 in the case of a shock, i.e., if $e_1^l = 0$. Observe that even when banks follow an imprudent lending policy, the proportion of firms q that fail under a shock is relevant. The reason is that a fraction $1 - q$ of entrepreneurs make profits in the first period under a shock, and this possibility has an effect on the demand for loans. We have that total expected profits for a bank at $t = 0$ are:

$$\Pi_0(l_0) = \begin{cases} \Omega^h(L)l_0 + (\beta p - 1)e_0 & \text{if } e_1^l = 0 \\ (\Omega^h(L) + \Omega^l(L))l_0 + \left((\beta p - 1) + \frac{\beta^2(1-p)}{\alpha_1} \left(\frac{y_2}{\lambda} - (1-\alpha_1) \right) \right) e_0 & \text{if } e_1^l > 0 \end{cases} \tag{15}$$

There are two points to make about this expression for bank profits. First, banks maximize profits subject to the capital adequacy restriction $l_0 \le e_0/\alpha_0$. If an *imprudent* equilibrium is chosen this condition is binding, because $\beta p - 1 < 0$, and therefore $\Omega^h(L)$ has to be strictly positive or the imprudent equilibrium would have negative profits. Since the *imprudent* equilibrium is linear in l_0, the capital constrain must be binding. Second, observe that the two profit functions are different and cannot be transformed into one another via a continuously differentiable transformation, because of the non-negativity constraint on profits if the bank collapses after a shock.

[18] Observe that

$$sgn\left(\frac{d\hat{q}}{dl_0}\right) = sgn\left(\frac{e_0}{l_0^2}(1 + r_0) - \frac{dr_0}{dl_0}\left(1 - \frac{e_0}{l_0}\right)\right) > 0.$$

Notation: We define the following notation which will be useful in the following:[19]

$$\Psi \equiv y_1 - \frac{G^{-1}(L^\star)}{p+(1-p)q} - \frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))},\ N\text{: number of banks.} \tag{16}$$

$$H \equiv \frac{\beta^2(1-p)}{\alpha_1}\left(\frac{y_2}{\lambda}-(1-\alpha_1)\right) \tag{17}$$

$$\phi \equiv 1-\beta(y_2-\lambda)+\lambda\alpha_1(1-\beta) \tag{18}$$

Observe that H is the contribution to profits of one additional unit of capital at time 1 in the bad state of the world, weighed by its probability of occurrence and discounted to time $t = 0$. We will use the following important assumption:

Assumption 3 $\beta p + H > 1$

The assumption means that the expected value of an additional unit of bank capital at $t = 1$, discounted to $t = 0$, is bigger than one, i.e., it is profitable on average to have more period 1 capital. To see this, observe first that

$$\frac{\beta}{\alpha_1}\left[\frac{y_2}{\lambda}-(1-\alpha_1)\right] > 1 \tag{19}$$

implies that in the bad state of the world the bank prefers to invest its remaining capital rather than not lend it. On the other hand, additional capital in the good state of the world is useless, since it is plentiful, and the excess may as well be paid out in dividends. Now note the assumption 3 can be written as:

$$\beta p + H = \beta p \cdot 1 + \beta(1-p)\left[\frac{\beta}{\alpha_1}\left(\frac{y_2}{\lambda}-(1-\alpha_1)\right)\right] > 1.$$

Since $\beta p < 1$, assumption 3 implies that equation (19) holds.

In order for imprudent equilibria to have a chance of being chosen, we must ensure that the *prudent* equilibria is interior to the capital adequacy constraint $l_0 = (e_0/\alpha_0)$ (otherwise there can be no *imprudent* equilibria , since they involve more lending than *prudent* equilibria). A sufficient condition for the *prudent* equiilibria to be interior to the capital adequacy constraint is that

$$\beta p\phi + H > 0$$

which we assume in the following. To see this, observe that

$$\frac{\partial \Pi^p}{dl_0} = 0 \Rightarrow \Psi = \frac{\beta p\phi + H}{\beta p + Hq}$$

in the Bertrand case, $\Psi = 1 + r_0$ so that if $\beta p\phi + H \leq 0 \Rightarrow 1 + r_0 \leq 0$ and in a *prudent* equilibrium

[19] Here v is the conjectural variation parameter, corresponding to the beliefs of firm i of its rivals' reaction to its own loan supply choices. We assume that v is identical for all firms. When $v = -1/(N-1), 0, 1$ we reproduce the Bertrand, Cournot and collusive equilibria.

lenders will always lend to the capital adequacy constraint. Under less competitive environments it will also be the case that the capital adequacy condition is not binding. We can rewrite this condition as

$$p\beta\phi + H = p\beta\left(1 - \beta(y_2 - \lambda) + \lambda\alpha_1(1 - \beta)\right) + H > 0. \tag{20}$$

which we use later. I

Another interpretation of the condition that ensures that *prudent* equilibria are interior is that projects are not so profitable that $1 + r_0 \le 0$, i.e., banks are unwilling to give away money in the first period under Bertrand competition.

5 Existence and uniqueness of equilibrium

As there are two potential profit functions, corresponding to *prudent* and *imprudent* behavior of banks, there are potentially two families of equilibria. We use the Pareto optimality criterion to choose among symmetric equilibria with the same starting capital e_0, and we show that there is a neighborhood of $p = 0$ in which the prudent equilibrium is chosen for all intensities of competition.[20] Our procedure is as follows: first, we show that there is a unique symmetric equilibrium to both *prudent* and *imprudent* behaviors by banks. Next we show that as competition decreases, the gap between the profits at the *prudent* equilibrium and the *imprudent* equilibrium increases. Then we show that under Bertrand competition, when $p = 0$ (i.e., the shock is a certainty) the *prudent* equilibrium has strictly positive profits while the *imprudent* equilibrium has negative profits. Hence the result continues to hold in the same neighborhood for lower intensities of competition.

Lemma 1 *There is a unique equilibrium of each type (*prudent, imprudent*) to the game among banks for any intensity of competition. In the non-Bertrand case we have*

$$\frac{\partial^2 \Pi_i}{\partial l_i^2} < 0 \quad \textit{and} \quad \frac{\frac{\partial^2 \Pi_i}{\partial l_i^2}}{\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j}} > 1$$

Proof: We examine the case of *prudent* equilibria here and the appendix contains the very similar analysis of *imprudent* equilibria. We begin by noting that the Bertrand case must be treated separately, because in that case L is presumed constant by banks. Hence, banks face a linear maximization problem, leading to *bang-bang* solutions in which either all banks do not lend (if $(\Omega^h(L) + \Omega^l(L)) < 0$) or they lend up to the capital adequacy constraint if the sign is positive. Interior solutions are possible only if $(\Omega^h(L) + \Omega^l(L)) = 0$. There is only one interior symmetrical equilibrium, since we require $Nl_0 = L$, where L satisfies

$$0 = \Omega^h(L) + \Omega^l(L)) = (\beta p + Hq)(1 + r_0) - (\beta p\phi + H)$$

[20] Optimality can be justified as a selection mechanism if communication is allowed or by evolutionary arguments.

Since $1+r_0$ is strictly decreasing in L, there is a single solution L and therefore a single symmetrical level of first period lending l_0 under Bertrand competition.

In non-Bertrand cases, the profit functions satisfy the standard conditions for existence and uniqueness of equilibria. We consider the case of *prudent* equilibria:

$$\frac{\partial \Pi_i}{\partial l_i} = (\beta p + Hq)\Psi - (\beta p \phi + H) \tag{21}$$

and thus

$$\frac{\partial^2 \Pi_i}{\partial l_i^2} = (\beta p + Hq)\frac{\partial \Psi}{\partial l_i} = -(\beta p + Hq)\left(\frac{2}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right) < 0$$

and:

$$\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j} = (\beta p + Hq)\frac{\partial \Psi}{\partial l_j} = -(\beta p + Hq)\left(\frac{1}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right)$$

from which we derive:

$$\frac{\frac{\partial^2 \Pi_i}{\partial l_i^2}}{\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j}} = \frac{\left(\frac{2}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right)}{\left(\frac{1}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right)} > 1$$

∎

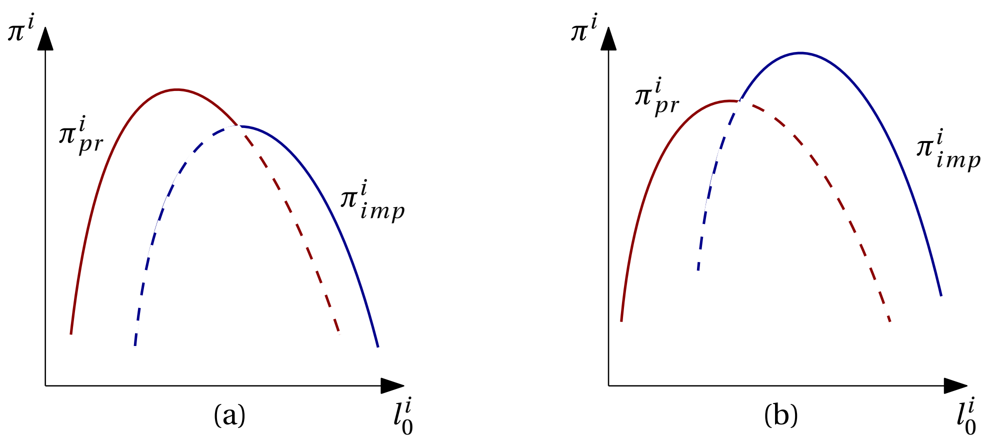

Figure 3: Equilibrium configurations

Consider diagram 3, which shows the possible configurations of the profit function for a single firm, given that the choices of the other firms are in a symmetric equilibrium. In general there will be two equilibria: a *prudent* equilibrium and an *imprudent* equilibrium. Each curve describes all possible deviations of the bank given what its rivals are choosing as their first period lending l_0 in the corresponding symmetric equilibrium. In figure (a) There is no incentive for the bank to jump to the lending associated to the imprudent equilibrium (given that the other firms are playing the prudent equilibria), since the point at which the curves cross is where $e_1^l = 0$ and its profits are lower by switching. In figure (b) the imprudent equilibrium is selected by the Pareto criterion. Note that prudent profits are not defined beyond the crossing, since $e_1^I < 0$ at those points and the prudent profit function is not defined there.

If the world were to resemble (b), then the role of regulation is to restrict l_0 so that the imprudent equilibrium cannot be attained. To see this last point, consider the case of figure 4. In the figure, the vertical line corresponds to the lending limit defined by the first period capital adequacy condition (13) and limits first period lending of any bank to that level, so that even though the *imprudent* equilibrium is preferred by banks, it cannot be chosen and firms prefer the *prudent* equilibrium to their other (symmetric) options.[21]

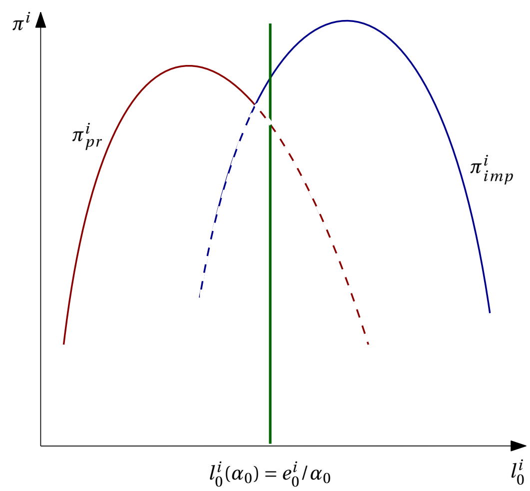

Figure 4: A prudential limit on overlending

The next step in the proof is to show that as competition decreases, the difference between the profits at the *prudent* and the *imprudent* equilibria increase.

Proposition 2 *As competition decreases, the* prudent *equilibrium becomes more attractive compared to the* imprudent *equilibrium (* $\frac{\partial(\Pi^P_\star - \Pi^I_\star)}{\partial v} > 0$*).*

Proof:: Assume first that the solution to both problems is interior. Then define the following optimization program:

$$W(x) \equiv \underset{l_0 \geq 0}{\text{Max}}\ x\Pi^P + (1-x)\Pi^I$$

which is the convex combination of the banks' problem in the two types of equilibrium. Rewriting, and using expressions 14–18 we obtain

$$\begin{aligned} W(x) \equiv \underset{l_0 \geq 0}{\text{Max}}\ & x\left[\beta p((1+r_0)-\phi)l_0 + H(q(1+r_0)-1)l_0 + (\beta p + H - 1)e_0\right] \\ & + (1-x)[\beta p((1+r_0)-\phi)l_0 + (\beta p - 1)e_0] \end{aligned}$$

[21]If the crossing between the two curves occurs to the left of the maximum of the curve corresponding to the *prudent* equilibrium, one cannot use the FOC to characterize the equilibrium. In this case a *prudent* equilibrium exists only if the capital adequacy restriction lies to the left of the crossing, where $(\partial\Pi^i_{pr}/\partial l^i_0) > 0$.

Using the Envelope Theorem and the definition of e_1^l from equation 6:

$$W'(x) = H[(q(1+r_0^\star(x)) - 1)l_0^\star(x) + e_0] \equiv He_1^{l\star}(x)$$

Thus:

$$\Pi_\star^P - \Pi_\star^I = W(1) - W(0) = \int_0^1 He_1^{l\star}(x)dx$$

By taking derivatives with respect to v we obtain:

$$\frac{\partial \Delta W}{\partial v} = H \int_0^1 \frac{\partial e_1^{l\star}(x)}{\partial v} dx > 0$$

We prove that $\frac{\partial e_1^{l\star}(x)}{\partial v} > 0$ $(\forall x)$. In effect:

$$\frac{\partial e_1^{l\star}(x)}{\partial v} = (q\Psi(x) - 1)\frac{\partial l_0^\star}{\partial v}$$

The derivative on the RHS is negative by proposition 4 below. For the other term in the RHS, note that from the FOC of the first period bank's problem,

$$[\beta p + Hqx]\Psi(x) = \beta p\phi + Hx$$

and thus

$$\Psi(x) = \frac{\beta p\phi + Hx}{\beta p + Hqx}$$

Finally, it is easy to check that:[22]

$$1 - q\Psi(x) = \frac{\beta p(1 - q\phi)}{\beta p + Hqx} > 0$$

finally, consider the case in which the capital adequacy conditions constrain lending in the *imprudent* equilibrium. Figure 5 shows the possible configurations and it is clear that the result continues to hold in this case. ∎

The intuition for this result is that with more competition a given shock leaves a bank with smaller values of second period capital in case of shock $(e_i^l(l_0^{Pr*}; q))$. This means that the prudent equilibrium is less attractive, since banks can finance fewer firms in the second period. The imprudent equilibrium, which foregoes financing firms in the second period in case of shock, becomes relatively more attractive.

The last stage in the proof is to find conditions under which the *prudent* equilibrium is preferable to the *imprudent* equilibrium in the Bertrand equilibrium. By proposition 2, this means that for any lower degree of competition, the *prudent* equilibrium continues to be chosen. More generally, if there is any level of competition for which under specified conditions the *prudent* equilibrium is preferred to

[22] This result is true when the FOC hold with equality. At $v = 0$ (Bertrand) there is the possibility of corner solutions. where the result does not necessarily hold, because solutions are of the bang-bang type.

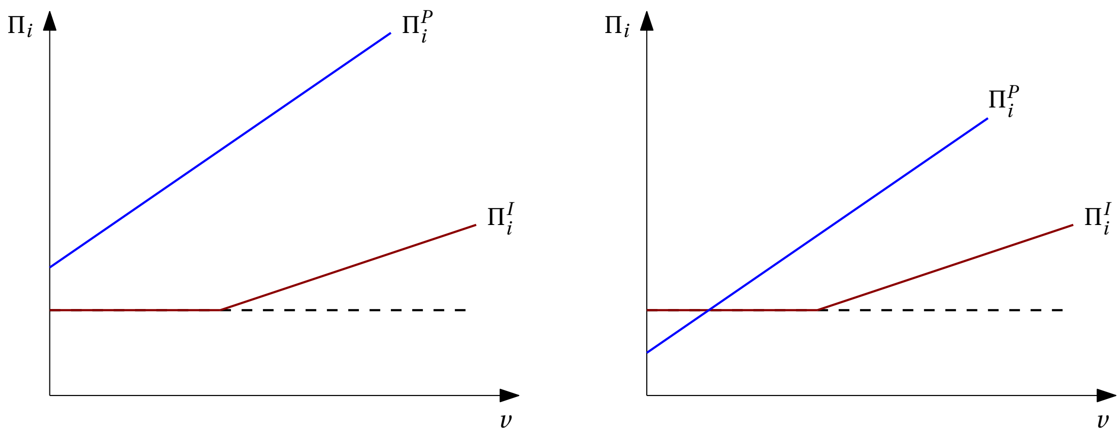

Figure 5: Profit differences increase when the imprudent equiilibrium is credit constrained for a range of competition parameters

the *imprudent* equilibrium, then this continues to hold true for any lower degree of competition.

The last result we need is to show that t there is a region in parameter space where *prudent* equilibria exist and are preferred to *ímprudent* equilibria.

Proposition 3 *There is a neighborhood of* $p = 0$ *in which* prudent *equilibria are preferred for all intensities of competition.*

Proof: We proceed by showing that there is a neighborhood where *prudent* equilibria are preferred in the Bertrand case, and therefore by proposition 2, will also be preferred in less competitive banking systems. Recall that under Bertrand competition, an interior equilibrium can exist only if $\Omega^h(L) + \Omega^l(L) = 0$. In that case, the bank's profits are

$$\Pi_B = (\beta p + H - 1)e_0 > 0,$$

The other case, in which $\Omega^h(L) + \Omega^l(L) > 0$ (if this term is negative there is no lending), leads to $l_0 = e_0/\alpha_0$ in the *prudent* equilibrium. This is the same amount of lending as in the *imprudent* equilibrium. Since the outcomes will be the same in the bad state, this is inconsistent with at least one of the two equilibria existing.

If $p = 0$, the shock always hits and in that case the *imprudent* equilibrium always leads to bankruptcy of the bank. In that case, we cannot be in a *prudent* equilibrium where lending is bound by the capital adequacy constraint since it would be inconsistent with the definition of a *prudent* equilibrium.

Thus only interior *prudent* equilibria are viable and we showed above that profits in the Bertrand *prudent* equilibrium are strictly positive. By continuity of the profit functions, there is a neighborhood of $p = 0$ in which *prudent* equilibria are also chosen. Note also that by proposition 4 , lower intensity of competition leads to less lending, and therefore lower probabilities of collapse.

∎

It is interesting to note that there is also a range for which the *imprudent* equilibria are preferred for any intensity of competition. Consider the case where $p \approx 1$ and $q \approx 0$, so there is a small probability of a shock, but when it occurs avoiding a banking collapse does not provide an advantage because most firms fail; $e_1^l > 0$ but small in the *prudent* equilibrium. In this case, it is easy to show that an *imprudent* equilibrium is preferred (in the limit, banks do not lend in the *prudent* equilibrium). Essentially, avoiding an improbable collapse, and not gaining much by it leads to an inefficient equilibrium, given the cost of constraining lending in the very probable state with no shock. This result becomes clearer when we examine the expressions for Expected GDP that are developed below.

6 Comparative statics for prudent equilibria (no bank failures)

Having shown the existence ranges of pareto selected *prudent* equilibria for certain parameter configurations, we can proceed to examine the comparative statics in this financial system. This corresponds to the case in which the economy is subject to relatively small shocks that do not endanger the banking system, or alternatively, that the banks are very well capitalized; or finally, that the value of the parameter α_0 restricts lending as in figure 4.[23]. In order to do comparative statics we note that Assumption 3 implies that the following holds:

$$p\beta[1 - \beta(y_2 - \lambda) + \lambda\alpha_1(1 - \beta)] + H > 0 \tag{22}$$

This inequality implies that in a prudent equilibrium under Bertrand competition (and thus for any less competitive scenario), first period lending is interior to the capital adequacy restriction given by α_0. We can now show the following important result:

Proposition 4 *Increased competition in banking (lower v) increases first period lending (and reduces the first period interest rate) in both prudent and imprudent equilibria. Moreover $e_1^l \downarrow$.*

Proof: Note that

$$\frac{\partial^2 \Pi_0}{\partial l_0 \partial v} = -\left(\frac{\beta p + \frac{\beta^2(1-p)}{\alpha_1}\left(\frac{y_2}{\lambda} - (1-\alpha_1)\right) q}{(p + (1-p)q)}\right) \frac{d}{dl_0}\left(\frac{l_0 \frac{\partial L}{\partial v}}{G'(G^{-1}(L))}\right)$$

where

$$\frac{d}{dl_0}\left(\frac{l_0 \frac{\partial L}{\partial v}}{G'(G^{-1}(L))}\right) = \frac{\frac{\partial L}{\partial v} + l_0 \frac{\partial^2 L}{\partial l_0 \partial v}}{G'(G^{-1}(L))} - \left(l_0 \frac{\partial L}{\partial v}\frac{\partial L}{\partial l_0}\right)\left(\frac{G''(G^{-1}(L))}{G'(G^{-1}(L))^3}\right) > 0$$

where the last inequality is implied by the fact that $G(\cdot)$ is increasing and concave. Thus profits have increasing differences in $(l_0, -v)$. We can use Topkis' Lemma, first period lending l_0^* is decreasing in v. For the case of *imprudent* equilibria, see the appendix. ∎

[23] In this last case we need, in addition, that $q < \hat{q}$ so the prudential equilibrium is viable, i.e., that the crossing of π_{pr}^i and π_{imp}^i occurs to the right of the highest point in π_{pr}^i. Otherwise the banking system is inherently unstable and we cannot perform comparative statics.

This result implies that a more competitive banking system leads to a more efficient economy, with increased economic activity in the first period. There are more entrepreneurs that carry out their projects given the lower interest rates. On the other hand, lending is riskier, because banks are more leveraged. In case of a shock, a larger fraction of the banks capital will be wiped out; that is, the banks loan book is riskier. [24] Thus, it is not clear that the expected second period product is higher as competition increases.

The expected value of GDP over the two periods is:

$$Y^P = p[(y_1 - 1) + (y_2 - \lambda)]l_0^\star + (1-p)[q(y_1 - 1)l_0^\star + \frac{e_1^{l\star}}{\lambda\alpha_1}(y_2 - \lambda)] + \int_{\bar{U}(r_0^\star)}^{G_{max}} u dG(u)$$

Note that the effect of an increase in the degree of competition among banks can be written as:

$$\frac{dY^P}{dv} = [P_e(y_1 - 1) + p(y_2 - \lambda)]\frac{dl_0^\star}{dv} + \frac{(1-p)}{\lambda\alpha_1}(y_2 - \lambda)\frac{de_1^{l\star}}{dv} - U(r_0^\star)G'(U(r_0^\star))\left(\frac{-\frac{dl_0^\star}{dv}}{P_e G'(G^{-1})(L^\star)}\right)$$

where the first and third terms are strictly positive, while the second term is negative, so the sign of the expression is ambiguous. The effect of competition on second period activity can be written as:

$$(y_2 - \lambda)\left[p\frac{dl_0^*}{dv} + \frac{(1-p)}{\lambda\alpha_1}\frac{de_1^*}{dv}\right]$$

Evaluating at the polar cases $p = 0, 1$ we see that when the risk of a shock is low ($p \approx 1$) increased competition is beneficial and raises second period activity by proposition 4 (and therefore GDP is unambiguously higher). On the other hand, when the risk of a shock is large ($p \approx 0$), increased competition decreases second period GDP.[25] We have:

Proposition 5 *When the risk of a shock is low, increased competition raises expected GDP and second period activity in a* prudent *equilibrium.*

Note that in the case $p = 1$ and $q = 0$, there is no leverage in the *prudent* equilibrium (i.e., $e_0 = l_0$) ad this provides strictly less GDP than the *imprudent* equilibrium, where banks do use leverage. Therefore, there is a neighborhood in which expected output is higher under *imprudent* equilibria. As mentioned in the previous section, the effort to avoid an outcome which is unlikely (a banking collapse following the shock), specially when the shock is very severe –and therefore few firms can pay back their loans– imposes too severe a constraint on lending, and it is preferable to risk the low probability shock.

The next result shows the risks associated to increased financial competition: it shrinks the range of shocks for which the prudent banking equilibrium is valid. Recall that in (7) the value $\hat{q}$ is the fraction

[24] Moreover individual loans are riskier for the bank: since the probability of renewing a loan is smaller when the fraction of the bank's capital that is lost increases, loans become riskier and therefore have a lower return for the bank, independently of the fact that interest rates are lower.

[25] Observe that in an *imprudent* equilibrium, there is second period activity only if there is no shock, and without a shock, competition increases activity. Therefore increased competition always increases expected GDP in *imprudent* equilibria.

of firm that survives a shock that leaves the banks on the threshold of failure in the case of a prudent equilibrium. An increase in $\hat{q}$ means that a smaller shock endangers the system. We have:

Lemma 2 *Increased banking competition (lower v) decreases the range of shocks ($q \in [\hat{q}, 1]$) for which the prudent equilibrium ($e_i^l(l_0^{Pr*}; q) > 0$) is well defined.*

Proof: From the definition in (7),

$$\hat{q} \equiv \frac{1 - \frac{e_0}{l_0^\star}}{1 + r_0(L(l_0^\star))} \tag{23}$$

By implicit derivation, we obtain:

$$\frac{d\hat{q}}{dv} = \left(\frac{e_0}{l_0^{\star 2}(1 + r_0)}\right)\frac{dl_0^\star}{dv} - \left(\frac{1 - \frac{e_0}{l_0^\star}}{(1 + r_0)^2}\right) r_0'(L^\star) N \frac{dl_0^\star}{dv} < 0$$

Because Proposition 4 shows that $\frac{dl_0^\star}{dv} > 0$.

∎

Next we show that even for prudent equilibrium, so there are no banking crisis, increased competition increases risk in the economy, because the magnitude of the "sudden stop" in lending in the second period after a shock is larger. Recall that $\theta \equiv (l_1^h / l_1^l)$ measures how much lending there is after a shock compared to lending without a shock, see (9) and that lending is directly associated to economic activity.

Proposition 6 *Consider a prudent equilibrium. Increased banking competition (lower v) leads to larger reductions in lending in the second period in the case of a shock.*

' **Proof:** In the equilibrium we have:

$$\theta = \frac{q\left(y_1 - \frac{G^{-1}(L(l_0^\star))}{p + (1-p)q}\right) - 1 + \frac{e_0}{l_0^\star}}{\lambda \alpha_1}$$

By implicit derivation we have:

$$\frac{d\theta}{dv} = \frac{\frac{dl_0^\star}{dv}}{\lambda \alpha_1}\left(\frac{qN}{G'(G^{-1}(L)(p + (1-p)q))} + \frac{e_0}{l_0^{\star 2}}\right) > 0.$$

∎

Note that this proposition, at its heart, has the notion that the equity of banks after a shock is smaller as competition increases. We show that a reduction in the size of the shocks, i.e., a reduction in risk, leads to higher first period lending, as expected.

Proposition 7 *Consider a prudent equilibrium. Assume that under Bertrand competition, first period lending is interior to the capital adequacy constraint. Then less risk (higher q) leads to more lending in the first period (higher l_0^*).*

Proof: The First Order Conditions of the first period maximization problem (21) imply:

$$(\beta p+Hq)\left(y_1-\frac{G^{-1}(L^\star)}{p+(1-p)q}-\frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))}\right)=H+\beta p\underbrace{\left(1+\lambda\alpha_1(1-\beta)-\beta(y_2-\lambda)\right)}_{\phi>0}>0,$$

where the sign is derived from Assumption 3. Hence, the term in the large parenthesis is positive,

$$\Psi\equiv y_1-\frac{G^{-1}(L^\star)}{p+(1-p)q}-\frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))}>0$$

Implicit differentiation of the First Order Conditions leads to (recall that the second term does not involve q)

$$\frac{dl_0^\star}{dq}=\frac{H\left(y_1-\frac{G^{-1}(L^\star)}{p+(1-p)q}-\frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))}\right)+(\beta p+Hq)\frac{(1-p)}{p+(1-p)q}\left(\frac{G^{-1}(L^\star)}{p+(1-p)q}+\frac{[1+(N-1)v]l_0^\star}{(p+(1-p)q)G'(G^{-1}(L^\star))}\right)}{(\beta p+Hq)\left(2\frac{[1+(N-1)v]}{(p+(1-p)q)G'(G^{-1}(L^\star))}-\frac{G''(G^{-1}(L^\star))[1+(N-1)v]^2l_0^\star}{(p+(1-p)q)G'^3(G^{-1}(L^\star))}\right)}$$

In this expression, the denominator is positive, so the sign of (dl_0^*/dq) is given by the sign of the numerator. Reorganizing terms, the numerator becomes:

$$\Psi\left\{\beta p(1-p)\left(\frac{\beta}{\alpha_1}\left(\frac{y_2}{\lambda}-(1-\alpha_1)\right)-1\right)\right\}+y_1(\beta p+Hq)\frac{(1-p)}{p+(1-p)q}$$

Since we have shown that $\Psi>0$ and all remaining terms in the numerator are positive.

∎

6.1 Regulatory forbearance

We have seen in section 4 that unexpected regulatory forbearance on the capital adequacy constraints in a anticipate regulatory forbearance after the shock. Note that this is equivalent studying the effect of reducing the value of the capital adequacy parameter α_1 in the bad state on the banker's problem at $t=0$.[26] We gather the results in the following proposition:

Proposition 8 *In a prudent equilibrium, unanticipated regulatory forbearance can dampen or eliminate the variance in second period outcomes. When regulatory forbearance is anticipated at $t=0$, the effect on the variance of second period outcomes is ambiguous and may even increase the variance of second period outcomes.*

[26](when there is no shock, the capital adequacy condition does not bind so relaxing it has no effect on second period behavior.

Proof: Recall that $\text{Var}(l_1) = p(1-p)(\lambda l_0 - l_1^l)^2$. From the definition of e_1^l in (6) we have that $e_1^l = q(1+r_0)l_0 - (l_0 - e_0)$, and $l_1^l = e_1^l/\alpha_1$. Therefore:

$$\frac{dl_1^l}{d\alpha_1} = \underbrace{-\frac{e_1^l}{\alpha_1^2}}_{\text{Static Effect}<0} + \underbrace{\frac{1}{\alpha}\frac{de_1^l}{dl_0}\frac{dl_0}{d\alpha_1}}_{\text{Strategic Effect}} \tag{24}$$

The static effect corresponds to an unexpected change in the capital adequacy parameter, and its sign is always negative or zero. The second term in the RHS corresponds to the changes induced by the knowledge that, in case of a shock, the regulator will exercise forbearance. Now

$$\begin{aligned} \frac{d\text{Var}(l_1)}{d\alpha_1} &= p(1-p)2(\lambda l_0 - l_1^l)\left(\lambda\frac{dl_0}{d\alpha_1} - \frac{dl_1^l}{d\alpha_1}\right) \\ &= p(1-p)2(\lambda l_0 - l_1^l)\left\{\underbrace{\left(\lambda - \frac{1}{\alpha}\frac{de_1^l}{dl_0}\right)}_{>0}\frac{dl_0}{d\alpha_1} + \frac{e_1}{\alpha_1^2}\right\} \end{aligned} \tag{25}$$

because $(de_1^l/dl_0) = q\Psi - 1 < 0$. We need to determine the sign of $(dl_0/d\alpha_1)$. In a *prudent* equilibrium

$$\frac{dl_0}{d\alpha_1} = \frac{H'(\alpha_1)(1-q\Psi) + \lambda\beta p(1-\beta)}{-(\beta p + Hq)\left(2\frac{[1+(N-1)v]}{(p+(1-p)q)G'(G^{-1}(L^\star))} - \frac{G''(G^{-1}(L^\star))[1+(N-1)v]^2 l_0^\star}{(p+(1-p)q)G'^3(G^{-1}(L^\star))}\right)}$$

where the denominator corresponds to the second order condition and is therefore negative. Thus the sign of the derivative $(dl_0/d\alpha_1)$ corresponds to the sign of

$$-H'(\alpha_1)(1-q\Psi) - \lambda\beta p(1-\beta).$$

Now, $-H'(\alpha_1)(1-q\Psi) > 0$, since $(1-q\Psi) > 0$ (at the optimum) and $H'(\alpha_1) < 0$. On the other hand, $-\lambda\beta p(1-\beta) < 0$ so in general the sign of $(dl_0/d\alpha_1)$ is ambiguous.

When $p \approx 0$, the second term is close to zero, while the first term is positive and bounded away from zero, implying that $dl_0/d\alpha_1 < 0$. Thus we have that when a shock is likely, the expectation of future regulatory forbearance in case of a shock increases first period lending.[27]

Conversely, when there is a low probability of shocks ($p \approx 1$), the sign of the numerator is strictly negative, thus $dl_0/d\alpha_1 > 0$. This means that expected future regulatory forbearance leads to reduced first period lending when the probability of a shock is low.

[27] The intuition is that when the probability of a shock is low, the capital adequacy constraint limits lending in the second period in the unlikely case of a shock, and the overall effect of expected regulatory forbearance is to increase lending in the first period.

Finally, consider the case $p = 1, q = 0$. Then, $e_1^l = 0$, $(de_1^l/dl_0) = q\Psi - 1 = -1$ and the numerator of $(dl_0/d\alpha_1)$ becomes $\lambda\beta(1-\beta) > 0$ so by equation (25),

$$\frac{dl_0}{d\alpha_1} < 0 \Rightarrow \frac{dVar(l_1)}{d\alpha_1} < 0.$$

Thus there is a neighborhood of $p = 1, q = 0$ where anticipated forbearance ($\alpha_1 \downarrow$) increases the variance of second period GDP.

∎

The model shows that anticipated forbearance has a strategic effect, altering the expected effect of forbearance: it encourages first period lending –less prudent behavior– and thus may increase overall GDP variance in the second period. Even when this paradoxical effect does of occur, it may still be true that the effect of regulatory forbearance on post-shock activity is dampened by the change in *ex ante* behavior induced by being anticipated. Finally observe that there is a neighborhood of $p = 0$ where anticipated forbearance reduces the variance of second period GDP.

7 Comparative statics between types of equilibria

We have been working under the assumption that the equilibrium do not involve a collapse of the banking system (i.e., we are not in an imprudent equilibrium). However, under certain conditions, a collapse of the banking system in the bad state of the world may be convenient for firms, because they do so much better in the good state of the world. That is, under certain conditions it may be convenient for bank owners to "bet the bank" on the non-occurrence of the bad state of the world. We have shown before that it is possible to use the capital adequacy conditions to exclude this possibility, forcing them to be more conservative. However, the regulator may not always apply these conditions, or the regulator may be incapable of supervising the bank's compliance with the rule. For this reason, we explore the case

Up to now we have restricted the analysis to comparatives statics around the prudent equilibria. However, competition may lead banks to become imprudent, so it is essential to analyze the case in which the option of the imprudent equilibrium is allowed. We can now proceed to the main result of the paper.

Proposition 9 *Increased banking competition always leads to increased variance in second period economic outcomes. This occurs within and among types of equilibria.*

Proof: Consider first the case of a prudent equilibrium. Proposition 6 shows that if we are in the range in which increased competition leads to a prudent equilibrium, so there is no switch to an *imprudent* equilibrium, second period economic results have increased variance.

Now consider *imprudent* equilibria. Since competition implies higher lending in these equilibria, economic activity without a shock is higher. On the other hand, when there is a shock, lending and the associated economic activity is always zero. Hence the variance of second period economic activity increases.

Finally, note that by proposition 2, when competition increases, the equilibria can go from *prudent* to *imprudent*, and never in the other direction. We can decompose the effects of increased competition and a switch in type of equilibria as an increase in competition among *prudent* equilibria, and a switch between types of equilibria, keeping constant the intensity of competition. The first effect increases the variance of second period economic activity. Furthermore, the jump from a *prudent* to an *imprudent* equilibrium, keeping constant the intensity of competition increases the variance of economic results, since loans are larger under the imprudent equilibrium and thus second period economic activity is higher when there is no shock, and the effects of the shock are also more severe.

∎

8 Extensions

In these sections we extend the results in various directions.

8.1 Competition and risk choice

There is a literature (see section 2) that shows that increased competition leads to less risky lending by banks. This countervailing effect can be included in our model. Consider that case where there are potentially two types of projects, differentiated by their risk. More precisely, assume that we have two types of projects a, b, such that $q_a > q_b$ but with the same expected value at $t = 1$ (at $t = 2$ they have the same behavior). Thus we must have that $q_a y_1^a = q_b y_1^b \Rightarrow y_1^a < y_1^b$.

Now, from the point of view of borrowers, both types of projects are equally attractive, if they could finance the projects themselves. since entrepreneurs receive returns only at $t = 1$ and they are protected by the bankruptcy law if the project fails. Consider now the case of banks. Lending to the riskier projects leads to worse results in the case of failure, because a higher proportion of these loans go into default. Now consider the case in which only the type a projects exist and suddenly a small fraction of type b projects appear (so that the initial equilibrium is not affected by the introduction of the new projects). Clearly in an environment with less competitive banks there will be relatively more interest in lending to these riskier projects, as they get a comparatively larger share of y_1^b in the good state of the world (but borrowers will be less interested in these projects if there is little competition).

8.2 Leverage

The results on stability depend on competition through its effects on the amount of leverage that banks choose. At a fundamental level, it is the increase in leverage that causes instability, and this raises the question of whether there are other policies that increase leverage and therefore tend to increase instability. Consider the following expamples.

8.2.1 Openess

Assume that banks have to pay a positive rate on deposits (in the previous model, this added nothing to the results). Now assume that a closed economy liberalizes its financial markets and savings can flow in or out of the economy. If the domestic rate on deposits previous to opening the economy was higher than the international rate, after opening bank leverage would increase. Moreover, because the pass through of a cost reduction is higher thewhe na market is more competitive, we have that the impact on leverage of opening the financial markets –and therefore of instability– is reinforced when th banking sector is more competitive.

8.2.2 Improved creditor protection

Consider improved creditor protection as described in Balmaceda and Fischer (2010). Assume that in the case that a firm does not obtain a second period loan, there is a residual value to the bank (but it cannot be used immediately for second period loans), and that this residual value increases with the quality of creditor protection. Thus leverage will increase, and again we will have more instability, in this case because of increased creditor protection.

9 Conclusions

This paper has examined the effects of increased banking competition in two period model where a first period shock to economic activity leads to defaults on loans. These defaults lower the capital and reserves of banks, reducing their lending in the second period. Thus the first period shock is amplified by the banking system. We study the effects of varying degrees of competition in this setting.

The model allows us to understand several phenomena in the interaction between banking competition, economic activity and regulation. We have shown that there are two types of symmetric equilibria, which we denote by *prudent* and *imprudent* equilibria. Equilibria of the first type amplify the initial shock but do not cause the collapse of the banking system and the breakdown of lending activity, as occurs in the second type of equilibria. Both types of equilibria can be Pareto Optimal under different circumstances, such as the prevalence of shocks and their magnitude.

We have a series of results related to the effects of increasing competition among banks. First, as competition increases, the *imprudent* equilibria become relatively more attractive to banks. Moreover, even when we consider only *prudent* equilibria, increased competition means that the amplification of the initial shock is larger, because banks tend to lend more and therefore a shock leads to more capital and reserves being used to pay back depositors. This leads to less lending in the second period, because banks are restricted by capital adequacy parameters. We also show that when the risk of a shock is low, increased competition raises GDP (in expectation), as well as expected second period activity.

The paper also examines the role of the banking regulator. In the model, capital adequacy rules can be used to exclude*imprudent* equilibria. This is consistent with the observation that required capital ratios have risen after the experience of the 2008 financial crisis. We also show that unanticipated regulatory forbearance in the aftermath of a shock can be used to reduce or even to eliminate the amplification effect. However, we also show that when banks predict that there will be regulatory forbearance after a shock, the effects of forbearance on economic activity are ambiguous. Paradoxically, it is possible that anticipated forbearance increases the variance of second period activity by encouraging first period lending and thus a larger amplification of the initial shock.

Our final result is to show that within and between types of equilibria, increased competition always leads to increased variance in second period economic activity.

A worthwhile extension of this approach would be to have entrepreneurs differentiated by their capital endowments, and have credit rationing driven by informational asymmetries or legal deficiencies. Such a model would allow us to study the interaction between the legal protection for lenders and the effects of banking competition, or the interaction between the distribution of wealth, competition and stability.

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A Appendix

Proposition 1 *At date $t = 1$, each bank makes loans of* $l_1^s = \min\left\{\frac{e_1^s}{\alpha_1}, \lambda l_0\right\}$ *and takes deposits of* $d_1^s = (1-\alpha_1^s)l_1^s$ *and pays dividends of* $\text{Div}^s = e_1^s - \alpha_1 l_1^s$.

Proof: Rearranging terms and using the equality condition $e_1^s + d_1^s - l_1^s = Div^s$ plus $1 + r_1 = \frac{y_2}{\lambda}$, the problem of the bank is:

$$\underset{l_1^s, Div^s}{Max}\ \beta\left(\frac{y_2}{\lambda} - 1\right)l_1^s + (1-\beta)Div^s + \beta e_1^s$$

$$\text{s.t.} \quad e_1^s \geq \alpha_1 l_1^s + Div^s$$

$$\lambda l_0 \geq l_1^s$$

Now, note that by Assumption 2 the first term in the objective function is positive. Moreover, Assumption 2 ensures that $\beta\left(\frac{y_2}{\lambda} - 1\right) > (1-\beta)$, so the objective function increases more with l_1^s than it increases with Div^s. Hence, Div^s is positive only if the second restriction is binding. Therefore, it is direct that in equilibrium:

$$l_1^s = \min\left\{\frac{e_1^s}{\alpha_1}, \lambda l_0\right\}$$

∎

Lemma 1 (*Imprudent* equilibria) *There is a unique equilibrium of each type (*prudent, imprudent*) to the game among banks, i.e.,*

$$\frac{\partial^2 \Pi_i}{\partial l_i^2} < 0$$

and

$$\frac{\frac{\partial^2 \Pi_i}{\partial l_i^2}}{\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j}} > 1$$

Proof: Case of imprudent equilibrium:

$$\frac{\partial \Pi_i}{\partial l_i} = \beta p(\Psi - \phi)$$

and thus

$$\frac{\partial^2 \Pi_i}{\partial l_i^2} = \beta p \frac{\partial \Psi}{\partial l_i} = -\beta p\left(\frac{2}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right) < 0$$

and

$$\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j} = \beta p \frac{\partial \Psi}{\partial l_j} = -\beta p\left(\frac{1}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right)$$

From which we derive:

$$\frac{\frac{\partial^2 \Pi_i}{\partial l_i^2}}{\frac{\partial^2 \Pi_i}{\partial l_i \partial l_j}} = \frac{\left(\frac{2}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right)}{\left(\frac{1}{P_e G'(G^{-1}(L))} - \frac{l_i G''(G^{-1}(L))}{P_e G'^3(G^{-1}(L))}\right)} > 1$$

■

Proposition 4 *We show that as competition increases in banking (lower v), first period lending increases in the case of an imprudent equilibrium.*

Proof: Note that:

$$\frac{\partial^2 \Pi_0^I}{\partial l_0 \partial v} = -\frac{\beta p}{(p+(1-p)q)} \frac{d}{dl_0}\left(\frac{l_0 \frac{dL}{dv}}{G'(G^{-1}(L))}\right)$$

Where,

$$\frac{d}{dl_0}\left(\frac{l_0 \frac{dL}{dv}}{G'(G^{-1}(L))}\right) = \left(\frac{\frac{dL}{dv} + l_0 \frac{d^2 L}{dl_0\, dv}}{G'(G^{-1}(L))}\right) - \left(\frac{l_0 \frac{dL}{dv} G''(G^{-1}(L))}{G'(G^{-1}(L))}\right)$$

Given that $G(\cdot)$ is assumed to be increasing and concave we conclude that the RHS of the expression above is strictly positive and therefore we get the result.

■